

DETERMINATION OF THE TEMPERATURE  
DEPENDENCE OF THERMOPHYSICAL CHARACTERISTICS  
OF SOLID MATERIALS

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A method is proposed to determine the thermophysical characteristics of solid materials on the basis of solving third- and fourth-order linear heat-conduction equations. It is shown that linear heat-conduction equations of order greater than two possess a high degree of accuracy of the solution.

The majority of methods to determine the temperature dependence of the thermophysical characteristics of solid materials is based on the solution of linear heat-conduction equations in which these characteristics are assumed constant. This evidently introduces definite errors which it is not possible to avoid.

Methods are known, [1, 2], for example, which are based on the solution of nonlinear heat-conduction equations. However, their practical use is fraught with definite difficulties, since functional dependences of the desired quantities enter in the computational formulas.

A method which is distinguished from those preceding by the higher degree of accuracy in determining the thermophysical parameters and by the convenience of practical application is proposed in this paper.

The crux of the method is the following: We represent the nonlinear equation of heat conduction

$$\rho(t)c(t) \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x} \left( \lambda(t) \frac{\partial t}{\partial x} \right) \quad (1)$$

as

$$\rho_0 c_0 \frac{\partial t}{\partial \tau} - \lambda_0 \frac{\partial^2 t}{\partial x^2} = \frac{\partial}{\partial x} \left( \varphi(t) \frac{\partial t}{\partial x} \right) - \psi(t) \frac{\partial t}{\partial \tau}, \quad (2)$$

where  $\lambda(t) = \lambda_0 + \varphi(t)$ ,  $\rho(t)c(t) = \rho_0 c_0 + \psi(t)$ .

Since the functions  $\varphi(t)$  and  $\psi(t)$  are continuous and have continuous first to n-th derivatives with respect to x inclusive, then the right side of (2) can be represented as the sum of a power series:

$$\rho_0 c_0 \frac{\partial t}{\partial \tau} = \lambda_0 \frac{\partial^2 t}{\partial x^2} + \Phi(x, \tau) + \frac{1}{1!} \Phi'_x(x_0, \tau) (x - x_0) + \dots + \frac{1}{(n-1)!} \Phi_{x^{n-1}}^{(n-1)}(x, \tau) (x - x_0)^{n-1} + \dots, \quad (3)$$

where  $\Phi(x, \tau) = (\partial/\partial x)[\varphi(t)(\partial t/\partial x)] - \psi(t)(\partial t/\partial \tau)$  and  $x_0$  is an arbitrary point of the domain of variation of the space coordinate x. After n-tuple differentiation of (3) with respect to x, we obtain a linear differential equation of heat conduction of (n + 2)-th order:

$$\frac{\partial^{n+1} t}{\partial x^n \partial \tau} = a_0 \frac{\partial^{n+2} t}{\partial x^{n+2}} \quad (4)$$

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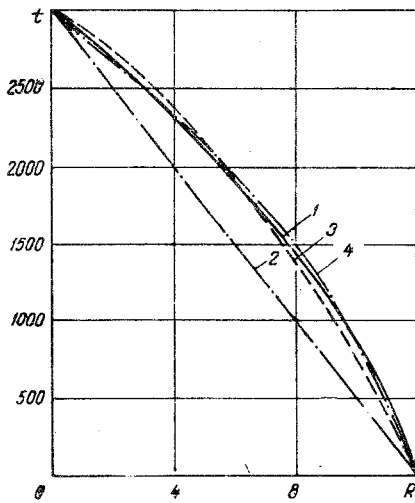


Fig. 1. Graphs of the temperature fields: 1) according to (8); 2) according to (9); 3) according to (10); 4) according to (11);  $t$  is in  $^{\circ}\text{C}$ ;  $R$  is in mm.

To solve (4) it is necessary to have  $n + 2$  boundary conditions, which can be given on both the boundary surfaces of the body and at points within it. Hence, it can be concluded for  $n \gg 1$  that the temperature field described by (4) practically agrees with the solution of (1). However, although (4) is linear, its solution for  $n \gg 1$  is also difficult. In this connection, it is expedient to find a minimal value of  $n$  such that the solution of (4) would agree with the exact solution of the nonlinear equation of heat conduction to a sufficiently high degree of accuracy. The simplest means of determining  $n_{\min}$  is to compare the solution of (4) for different values of  $n$  with the exact solution of (1). In this case, let us limit ourselves to solutions of just the stationary equations of heat conduction.

Let the nonlinear stationary heat-conduction equation

$$\frac{d}{dx} \left[ (\lambda_0 + \lambda_1 t_1) \frac{dt_1}{dx} \right] = 0 \quad (5)$$

with the boundary conditions

$$t_1|_{x=0} = t_0, \quad (6)$$

$$t_1|_{x=R} = 0. \quad (7)$$

be given. Then the solutions of (5) and (4) are for  $n = 0, 1, 2$ , respectively;

$$t_1(x) = -\frac{\lambda_0}{\lambda_1} + \frac{1}{\lambda_1} \left[ \lambda_0^2 + 2\lambda_1 \frac{R-x}{R} (\lambda_0 t_0 + 1/2 \lambda_1 t_0^2) \right]^{1/2}, \quad (8)$$

$$t_2(x) = t_0 \frac{R-x}{R}, \quad (9)$$

$$t_3(x) = t_0 \frac{R-x}{R} + [2t_0 - 4t_1(1/2R)] \frac{x^2 - Rx}{R^2}, \quad (10)$$

$$t_4(x) = t_2(x) + 9/2 [3t_1(1/3R) - 3t_1(2/3R) - t_0] \times \\ \times \frac{x^3 - R^2x}{R^3} + 9/2 [4t_1(2/3R) - 5t_1(1/3R) + 2t_0] \frac{x^2 - Rx}{R^2}, \quad (11)$$

where  $t_1(1/2R)$ ,  $t_1(1/3R)$ , and  $t_1(2/3R)$  are additional boundary conditions taken from the solution of (8). Graphs of the functions of the temperature fields corresponding to the solutions (8)-(11) are represented in the figure for  $\lambda = 0.2 \text{ W/(m} \cdot \text{deg)}$ ,  $\lambda_1 = 0.002 \text{ W/(m} \cdot \text{deg)}^2$ ,  $t_0 = 3000^{\circ}\text{C}$ , and  $R = 0.012 \text{ m}$ . It follows from an analysis of the curves that the solution (10) and even more so (11) assure an accuracy perfectly adequate for practice. Taking into account the temperature drops at  $3000^{\circ}\text{C}$  in solving (4) and (5), an analogous conclusion can be made relative to the solution of nonstationary heat-conduction equations of corresponding orders. Therefore, linear equations of the third or fourth order, whose solution will not raise any difficulties, can be used in developing some method to determine the thermophysical parameters of solid materials with an insignificant loss of accuracy. Thus, for example, in the case of symmetric heating of a plate of thickness  $2R$  and of given boundary conditions

$$t|_{\tau=0} = t_0, \quad (12)$$

$$t|_{x=0} = t_0 + \varphi(\tau), \quad (13)$$

$$t|_{x=R} = t_0 + \psi(\tau), \quad (14)$$

$$\left. \frac{\partial t}{\partial x} \right|_{x=0} = 0. \quad (15)$$

the solution of (4) for  $n = 1$  will be

$$t(x, \tau) = t_0 + \varphi(\tau) + (\psi(\tau) - \varphi(\tau)) \frac{x^2}{R^2} + (\psi'(\tau) - \varphi'(\tau)) \frac{x^4 - R^2x^2}{16a_0R^2}, \quad (16)$$

if it is a question of just the first-order derivatives of the functions  $\varphi(\tau)$  and  $\psi(\tau)$ . In (16)  $a_0$  is the coefficient of thermal diffusivity corresponding to the temperature  $t_0$  and assumed known. It follows from an analysis of the nonlinear heat-conduction equation (1) as  $x \rightarrow 0$  that

$$\rho(t)c(t) \frac{\partial t}{\partial \tau} \Big|_{x=0} = \lambda(t) \frac{\partial^2 t}{\partial x^2} \Big|_{x=0}$$

or

$$a(t) \Big|_{x=0} = \left[ \frac{\partial t}{\partial \tau} \left( \frac{\partial^2 t}{\partial x^2} \right)^{-1} \right] \Big|_{x=0}$$

Taking account of the above and the solution (16), we obtain the following formula to determine the coefficient of thermal diffusivity:

$$a(t) \Big|_{x=0} = \frac{\varphi'(\tau) R^2}{2(\psi(\tau) - \varphi(\tau)) - (\psi'(\tau) - \varphi'(\tau)) \frac{R^2}{8a_0}} \quad (17)$$

A formula can also be obtained in an analogous manner from the solution of a fourth-order linear heat-conduction equation. Thus, if the boundary condition

$$t \Big|_{x=R_1} = t_0 + g(\tau)$$

is added to the system of equations (12)-(15), where  $0 < R_1 < R$ , then the desired formula will be

$$a(t) \Big|_{x=0} = \frac{\varphi'(\tau) R(R - R_1)}{F(\tau) - \Phi(\tau)} \quad (18)$$

where

$$F(\tau) = (g(\tau) - \varphi(\tau)) \frac{2R^2}{R_1^2} - (\psi(\tau) - \varphi(\tau)) \frac{2R_1}{R},$$

$$\Phi(\tau) = (g'(\tau) - \varphi'(\tau)) \frac{R^2(3R_1 - 2R)}{30a_0R_1} - (\psi'(\tau) - \varphi'(\tau)) \frac{R_1(3R - 2R_1)}{30a_0}.$$

It is not difficult to determine the heat-conduction coefficient if the heat flux on at least one of the boundary surfaces of the body being tested is given. In principle, (17) and (18) differ from the well-known formula

$$a = \frac{bR^2}{2\Delta t} \quad (19)$$

Firstly, (19) has been obtained from the solution of a second-order linear equation of heat conduction and admits of its application for temperature drops on the order of 10-15°C between the boundary surfaces. Secondly, when it is used, the question remains open as to what temperature the values obtained refer (at the center, on the surface, or in the middle). As regards (17) and (18), they have been obtained from the solution of third- and fourth-order linear heat-conduction equations, respectively; therefore, their accuracy will be greater. Moreover, they are without that uncertainty which was expressed relative to (19), since the thermal diffusivity is determined at the point  $x = 0$  according to (17) or (18). Finally, they do not limit the temperature drop between the boundary surfaces.

#### LITERATURE CITED

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